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SUMMARY

1. PURPOSE. To provide security and policy review on the document at Tab 1 prior to release to the public.

2. BACKGROUND.
Authors: KIMBERLY ALBANESE, MARY MEYERS, KASSANDRA PRUSKO, AND MICHAEL COURTNEY

Title: Altitude Dependence of Rocket Motor Performance

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Michael Courtney, PhD
QRC Director/DFRL

1 Tabs
1. Altitude Dependence of Rocket Motor Performance

Altitude Dependence of Rocket Motor Performance

KIMBERLY ALBANESE, MARY MEYERS, KASSANDRA PRUSKO, AND MICHAEL COURTNEY

United States Air Force Academy,[1] 2354 Fairchild Drive, USAF Academy, CO, 80840

Michael.Courtney@usafa.edu

Abstract: It is well known that rocket motors produce greater thrust at higher altitudes due to the lower ambient pressure. A simple formula predicts that the thrust increase will be equal to the decrease in ambient pressure times the cross sectional area of the throat of the nozzle. This paper provides experimental tests of predicted increases in impulse based on this simple formula. The experimental method employed 20 trials with each of three solid fuel hobbyist rocket motors: Estes A10-PT, Estes D5-P and Quest D11-0P. Thrust curves were measured with a force plate for ten trials with each motor at two different altitudes corresponding to ambient pressures of 22.80 in Hg and 29.84 in Hg. Resulting impulses and specific impulses were then computed for each trial and the differences in average impulse and specific impulse were then compared with the predictions using the simple model that predicts thrust dependence on ambient pressure. Although thrust increases were measured in each case of reduced ambient pressure, the gains in impulse were not in agreement with predictions of the simple formula in every case.

Keywords: *rocket motor, thrust, impulse, specific impulse, ambient pressure*

Introduction

Performance of rocket motors can be quantified employing metrics based on the measured curve of thrust vs. time (Benson, 2008; Sutton and Biblarz, 2010). The area under the thrust curve is the impulse, and the impulse divided by the fuel mass is the specific impulse. The built up pressures inside the combustion chamber exit through a nozzle designed to increase pressure in the combustion chamber and lead to high velocity gases exiting the nozzle. Thrust is the force that propels the rocket (Benson, 2008). As a rocket is fired, the thrust is measured over the duration of the burn.

Relating the theoretical change of performance to change in ambient pressure requires consideration of the rocket thrust equation,

$$F=\frac{dm}{dt}v+\left(P_e-P_a\right)A$$, (Equation 1)

where:

F = net rocket engine thrust (N),

dm/dt = mass flow rate of exhaust gas (kg/s),

v = exhaust gas velocity at nozzle exit (m/s),

P_e = exhaust gas pressure at nozzle exit (Pa),

P_a = external ambient pressure (also known as free stream pressure, Pa), and

A = cross-sectional area of nozzle exhaust exit (m²).

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The change in thrust when firing the same rocket motor design at two different ambient pressures, P_h (higher pressure) and P_l (lower pressure) is then

$$\Delta F = \left(P_h - P_l\right) A \quad \text{. (Equation 2)}$$

Integrating over time gives the change in impulse,

$$\Delta I = \int \Delta F dt = \left(P_h - P_l\right) A t_{burn} \quad \text{, (Equation 3)}$$

where t_{burn} is the duration of the rocket firing. This paper presents the results of experimentally testing the predictions of equation 3 regarding the expected change in impulse and specific impulse with ambient pressure.

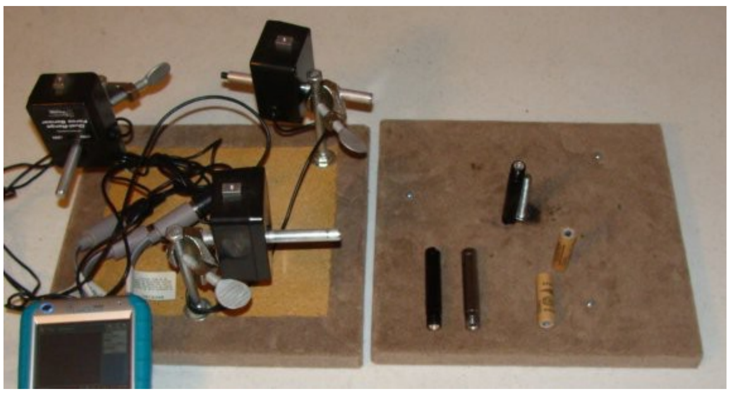

Figure 1: Force plate for measuring rocket engine thrust curves. The rocket motor is attached to the bolt extending out of the plate on the right which is set on top of the three force sensors on the left. (The photo shows one aluminum cased rocket motor attached to the bolt and two other aluminum cased motors in on the lower left corner of the plate, as well as two brown paper cased Estes A10-PT rocket motors.)

Method

The experimental method employed 20 trials with each of three solid fuel hobbyist rocket motors: Estes A10-PT, Estes D5-P and Quest D11-0P. Thrust curves were measured with a force plate for ten trials with each motor at two different altitudes corresponding to ambient pressures of 77.878 kPa and 101.04 kPa. The thrust measurement system has been described in detail previously (Courtney and Courtney, 2009). The force plate employs three Vernier “dual-range force sensors” connected to a Vernier LabQuest. The three force sensors are attached to a bottom plate as shown in Figure 1, and a force plate rests on top of them and has a bolt to which the rocket engine is attached for static thrust testing. (The plate is held in place by gravity, there is no adhesive or connectors.) The total thrust is the sum of the three individual force readings. (The force plate is zeroed after the motor is attached.) Each force sensor has a selectable range of either 10N or 50N, so that if the plate was perfectly balanced, the full scale would be either 30N or 150N minus the static load, depending on the sensor setting. However, since the plate is not perfectly balanced, the three force readings are not equal, and the full scale ranges are closer to 20N and 100N for the 10N and 50N sensor settings, respectively. This system design is capable of measuring thrust curves for the full range (1/4A to E) of commonly available hobby rocket engines, as well as many experimental rocket motor designs.

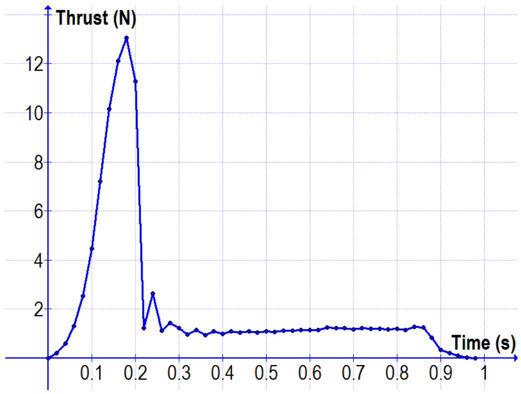

Figure 2: Thrust curve for Estes A10-PT Engine at low altitude (higher ambient pressure).

Figure 2 shows the measured thrust curve for an Estes A10-PT model rocket engine fired at low altitude. The shape of the thrust curve compares favorably with the published curve from the manufacturer. However, our experimental curve has a total impulse of 2.08 N s, compared with the manufacturer's claims of a total impulse of 2.5 N s. After the thust curve is measured with the force plate, resulting impulses and specific impulses were then computed for each trial and the difference in average impulse and specific impulse were then compared with the predictions using the simple model that predicts thrust dependence on ambient pressure. Fuel masses were computed by weighing each motor before and after firing, with the fuel mass determined as the loss of mass during firing.

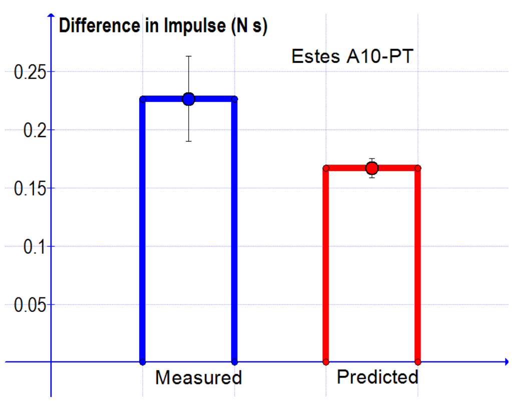

Figure 3: The measured and predicted differences between average impulses of the Estes A10-PT at high and low ambient pressures.

Results

Figure 3 compares the measured and predicted differences in average impulses for the Estes A10-PT at high and low ambient pressures. Equation 3 predicted that the mean impulse would be 0.167±0.008 N s greater at the lower ambient pressure of higher altitude. The uncertainty in the prediction comes from combining uncertainties in the throat area, burn duration, and ambient pressures. However, the mean impulse at high altitude was 2.232±0.029 N s, and the mean impulse at low altitude was 2.006±0.022 N s, where the uncertainty is the standard error of the mean. This is a measured difference of 0.226±0.037 N s. The predicted difference in impulse is in the right ballpark, but outside of the experimental uncertainties.

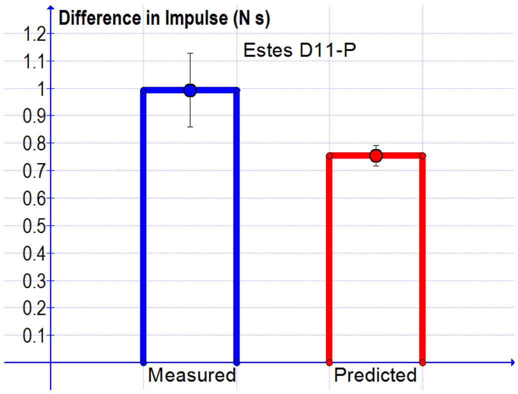

Figure 4: The measured and predicted differences between average impulses of the Estes D11-0P at high and low ambient pressures.

Measured and predicted differences in average impulses for the Estes D11-P at high and low ambient pressures are shown in Figure 4. Equation 3 predicted that the mean impulse would be 0.754±0.038 N s greater at the lower ambient pressure of higher altitude. However, the mean impulse at high altitude was 18.308±0.103 N s, and the mean impulse at low altitude was 17.315±0.087 N s. This is a measured difference of 0.993±0.135 N s. The predicted difference in impulse is in the right ballpark, but outside of the experimental uncertainties.

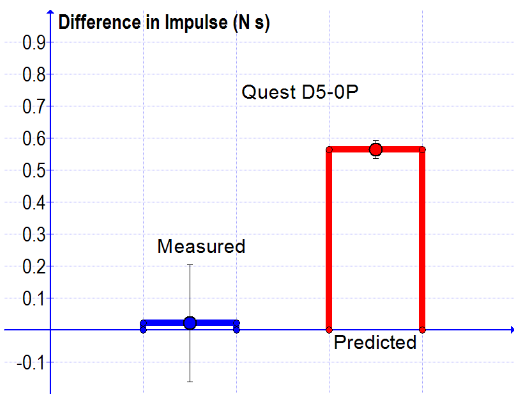

Figure 5: The measured and predicted differences between average impulses of the Quest D5-0P at high and low ambient pressures.

Figure 5 shows measured and predicted differences in average impulses for the Quest D5-0P. Equation 3 predicted that the mean impulse would be 0.564±0.028 N s greater at the lower ambient pressure of higher altitude. However, the mean impulse at high altitude was 19.245±0.106 N s, and the mean impulse at low altitude was 19.226±0.148 N s. This is a measured difference of 0.021±0.182 N s. The measured difference in impulse is much smaller than the predicted difference.

Discussion

In the two cases of the Estes rocket motors, the predictions of Equation 3 were relatively close, though outside the estimated uncertainties. The actual increase in impulse was 20-25% larger than might have been expected based on the simple formula based on the differences in ambient pressures, the cross sectional area of the throat, and the duration of the fuel burn. It may be that the effective area of the pressure differential is slightly bigger than the throat, or it may be that the assumptions that went into equation 3 are are not valid. The difference in ambient pressure might effect the mass flow rate or the exhaust velocity (the first term in Equation 1).

The discrepancy between the predicted and observed increases in impulse is larger and harder to explain for the Quest D11-0P. This rocket motor does not demonstrate any significant improvement in performance at the lower ambient pressure corresponding to higher altitude. It may be relevant that this rocket motor has a much longer burn time (4.6 seconds), a smaller throat in the nozzle (2.5 mm), and probably a much lower chamber pressure than the other motors. This motor also burns much louder and seems to sputter while burning which suggest a larger deviation from laminar flow.

It is also notable that all of these rocket motors fail to meet their published impulse specifications. The Estes A10-PT advertises an impulse of 2.5 N s, but actually achieves mean impulses of 2.232±0.029 N s and 2.006±0.022 N s, at high altitude and sea level, respectively. The Estes D11-P advertises an impulse of 20.0 N s, but actually achieves mean impulses of 18.308±0.103 N s and 17.315±0.087 N s at high altitude and sea level, respectively. The Quest D5-0P advertises an impulse of 20.0 N s, but actually achieves mean impulses of 19.266±0.148 N s and 19.245±0.106 N at high altitude and sea level, respectively. The present study is not the first to report hobby rocket motors failing to meet their impulse specifications. Measurements reported by a group at the University of Central Arkansas found that Estes rocket motor models A3-4T, A8-3, B4-4, B6-4, C6-5, and D12-3 have measured impuleses from 15.4% to 22.8% below the specifications (Penn et al., 2010). A study at the Australian Defense Force Academy found Estes D11-P motor to have an impulse 11.4% below Estes specifications and the C6-0 motor to be 4.45% below Estes' specifications (Carter, 2008), and an earlier study found the Estes A10-PT to be 20% below Estes' impulse specification.

The increase in thrust at higher altitudes is just one factor that will tend to enhance rocket performance in thinner atmospheres. Reduced drag also plays an important role. These two effects combine so that rockets launched at higher altitudes will be expected to fly higher, faster, and farther than rockets launched at sea level. Equation 3 may be a good first approximation to estimating enhanced performance at higher altitudes, but clearly other factors must be considered for accurate predictions.

Acknowledgements
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Appendix: Data

Test	Low Altitude Impulse Ns	Low Altitude Specific Impulse s	High Altitude Impulse Ns	High Altitude Specific Impulse s
1	2.080	57.605	2.262	62.658
2	2.012	58.113	2.368	68.381
3	1.985	56.679	2.112	60.305
4	1.958	54.674	2.389	66.720
5	2.023	54.837	2.219	60.159
6	1.953	53.387	2.194	59.960
7	2.171	59.166	2.144	58.436
8	1.990	54.822	2.213	60.969
9	1.915	56.918	2.190	65.085
10	1.973	54.790	2.323	64.523
Average	2.006	56.099	2.232	62.519
Std. Dev.	0.069	1.762	0.093	3.305
Uncertainty	0.022	0.557	0.029	1.045

Table A1: Impulse and specific impulse data for each rocket firing of the Estes A10-PT at low and high altitude.

Test	Low Altitude Impulse Ns	Low Altitude Specific Impulse s	High Altitude Impulse Ns	High Altitude Specific Impulse s
1	17.522	72.458	18.810	77.657
2	16.894	70.839	18.420	75.704
3	17.395	71.702	17.870	74.678
4	17.552	72.849	18.250	75.388
5	17.516	71.822	18.020	74.939
6	17.706	72.868	18.500	76.149
7	17.119	70.736	18.660	76.613
8	17.457	72.220	18.150	76.003
9	17.092	70.740	18.090	74.582
10	16.894	70.090	17.860	74.358
Average	17.315	71.632	18.308	75.746
Std. Dev.	0.276	0.932	0.326	1.035
Uncertainty	0.087	0.295	0.103	0.327

Table A2: Impulse and specific impulse data for each rocket firing of the Estes D11-P at low and high altitude.

Test	Low Altitude Impulse Ns	Low Altitude Specific Impulse s	High Altitude Impulse Ns	High Altitude Specific Impulse s
1	19.155	79.601	19.580	80.280
2	19.410	79.814	19.530	81.458
3	19.303	78.738	18.870	78.057
4	19.431	80.944	19.580	79.821
5	19.465	81.285	18.500	77.472
6	19.505	79.277	19.690	81.727
7	18.539	76.976	18.980	78.454
8	19.544	78.996	18.820	77.758
9	18.687	77.246	19.890	80.726
10	19.410	79.049	19.220	79.327
Average	19.245	79.193	19.266	79.508
Std. Dev.	0.334	1.303	0.468	1.608
Uncertainty	0.106	0.412	0.148	0.509

Table A3: Impulse and specific impulse data for each rocket firing of the Quest D50P at low and high altitude.